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A Note on a Theoretical Model for Economic Development

- Mathematical Representation of Lewis Model -

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Summary

This paper explores a theoretical model for economic development that presented by Lewis (1954), in particular, develops its mathematical representation.. A two-sector dual-economy model will be considered based on a Lewis model. This model consists of two sectors, such as a traditional subsistence sector and a modern capitalist sector. This model is based on Lewis (1954) and represented mathematically. According to this analysis following points are important for economic take-off in developing countries:

- (1) promotion of capital accumulation, in particular, that of high productivity.
- (2) encouragement of smooth labor shift from subsistence sector to capitalist sector with modest population growth.

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1. Introduction

After the currency crisis in 1997-98, southeast Asian countries appear recovering its economy with an engine of foreign direct investment. The key factor for economic development in this region is capital accumulation.¹

This paper intends to explore a theoretical model for economic development and to find some implications to Indonesia, which experienced the largest social and economic turmoil during the currency crisis. Lewis (1954) originally presented this type of dual-economy model and a lot of economists have been developing it including Banerjee and Newman (1996) and Freeman and Lindauer (1999). Emerson (2002) includes a factor of corruption in a dual-economy model. Solow (1956) and Goodfriend and McDermott (1995) contribute to theoretical aspect to a great deal.

This paper consists of five chapters other than this introduction; the following second chapter presents some assumptions for the model; the third chapter focuses on a theoretical framework of the model; the fourth chapter mathematically analyzes the model; and the final and fifth chapter concludes the discussion in the paper to derive some policy implications for both economic and Official Development Aid (ODA).

2. Basic Features

Some basic features, including assumptions, of Lewis model are as follows:

- (1) An economy consists of two sectors such as capitalist and subsistence sectors.
- (2) At the capitalist sector, wage rate is equal to marginal productivity of labor.
- (3) At the subsistence sector, wage rate is equal to subsistence level while marginal productivity of labor is nearly zero.²
- (4) All of capital stock exists only at the capitalist sector while none utilized at the subsistence sector.
- (5) The capital accumulation or investment originates from benefit of capital while wages at both sectors are consumed totally.
- (6) The capitalist sector goods are utilized for both consumption and investment while the subsistence goods are only for consumption.
- (7) Huge disguised unemployment exists at the subsistence sector, which could be a nearly infinite resource for unskilled labor supply for the capitalist sector.³
- (8) Prices of the subsistence sector goods are utilized as a measuring standard for the wage rate while the capitalist goods play the same role for subsistence sector goods.
- (9) A model itself is not dynamic but based on a comparative-static basis.
- (10) A closed economy is assumed for simplification of the model.⁴

3. Theoretical Framework

Lewis model then could be mathematically represented as follows:

¹ Mankiw *et al.* (1992) and Rappaport (2002) strongly insist of this point.

² Jorgenson (1961) implies that this assumption is not necessarily required for development analysis.

³ Sen (1966) deals with some example without labor surplus.

⁴ Later, foreign direct investment could be included as a resource of investment. This paper, however, excludes it.

(EQ1) production functions⁵

$$\begin{aligned} Y &= Y_C + Y_S \\ Y_C &= f(K, L_C) \\ Y_S &= g(L_S) \\ L &= L_C + L_S \end{aligned}$$

(EQ2) wage rates⁶ and aggregated wage incomes

$$\begin{aligned} w_c &= \frac{1}{p} \cdot \frac{\partial f}{\partial L_C} \\ w_s &> \frac{\partial g}{\partial L_S} = 0 \\ w_c &> w_s \\ W &= W_C + W_S \\ W_C &= w_c L_C \\ W_S &= w_s L_S \end{aligned}$$

(EQ3) capital accumulation⁷

$$\frac{dK}{dt} = rK - \mu K$$

or

$$K = \int_{-\infty}^0 (rK - \mu K) dt$$

(EQ4) demand functions

$$\begin{aligned} D &= D_C + D_S \\ D_C &= \phi[p, r, W_C, W_S, Y_S - W_S] \\ D_S &= \psi[p, W_C, W_S, Y_S - W_S] \end{aligned}$$

(EQ5) coincidence of demand and supply

$$\begin{aligned} Y &= D \\ Y_C &= D_C \\ Y_S &= D_S \end{aligned}$$

where Y_i output ($i=C, S$ or none)
 f, g production functions
 K capital stock (only at capitalist sector)
 L_i labor ($i=C, S$ or none)
 w_i wage rate ($i=C$ or S)

⁵ The output at subsistence sector depends only on labor.

⁶ The wage rate at both sectors is measured by the subsistence sector goods prices according to the assumption (7).

⁷ *ditto* as footnote 2.

W_i	wage income ($i=C, S$ or none)
p	price level of subsistence goods measured by capitalist goods
r	profit rate of capital at capitalist sector or marginal productivity of capital
	$r = \frac{\partial f}{\partial K}$
μ	depreciation rate of capital
t	time
D_i	demand ($i=C, S$ or none)
ϕ, ψ	demand functions

4. Mathematical Analysis

Based on the former theoretical model, we can develop it in a theoretical and mathematical manner.

Supposed that the labor supply is constant in a short run, the labor shift from the subsistence sector to the capitalist sector brings following four results:

- (1) decrease of subsistence sector goods output.
- (2) increase of wage income at the capitalist sector.
- (3) decrease of wage income at the subsistence sector.
- (4) increase of total wage income.

These results are mathematically represented as follows:

(EQ8) effect of labor shift from the subsistence sector to the capitalist sector

$$\frac{\partial g}{\partial L_C} < 0 \quad \text{since} \quad \frac{\partial g}{\partial L_S} > 0 \quad \text{and} \quad \frac{\partial L_S}{\partial L_C} < 0$$

$$\frac{\partial W_C}{\partial L_C} > 0$$

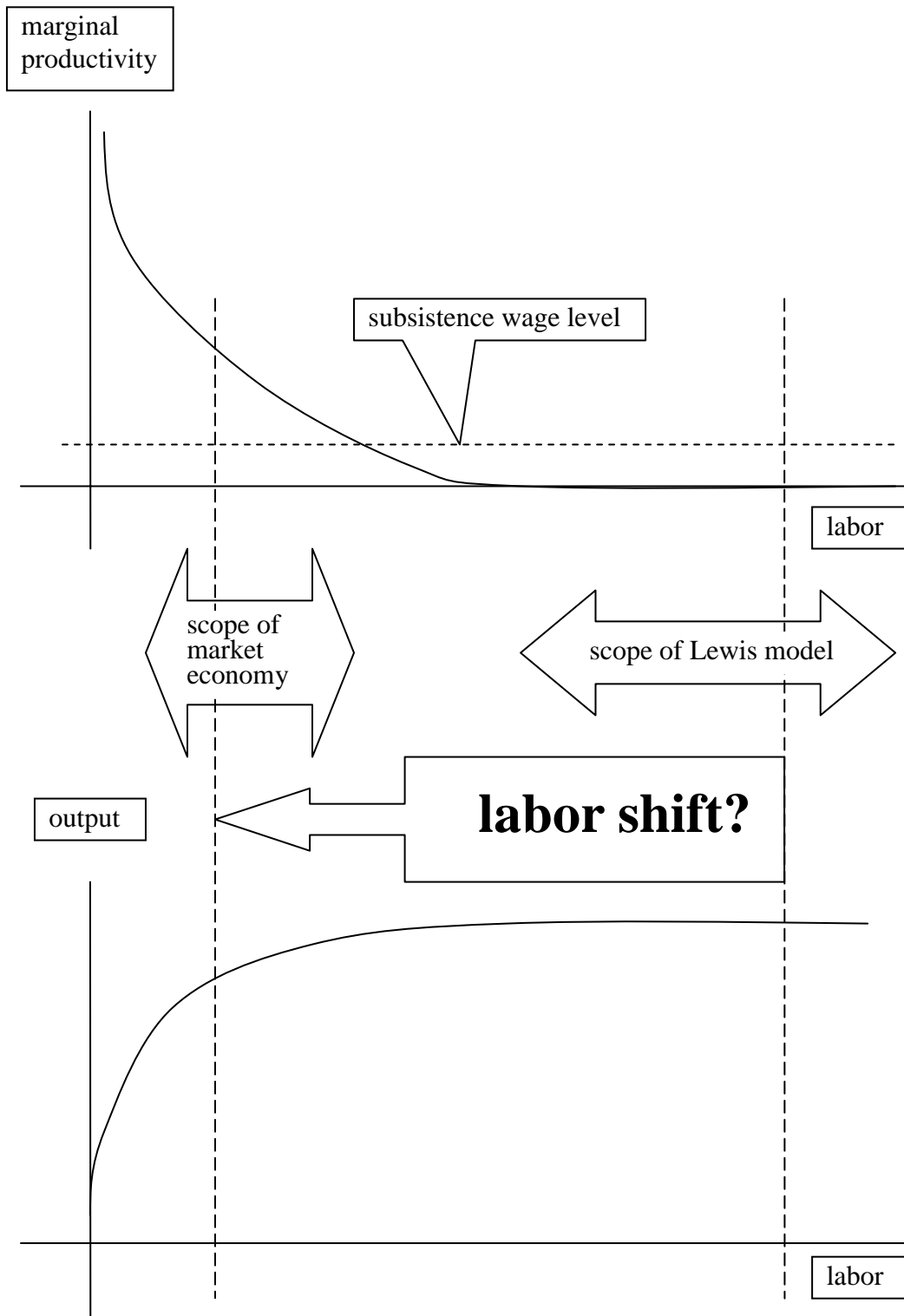
$$\frac{\partial W_S}{\partial L_C} < 0$$

$$\frac{\partial W}{\partial L_C} > 0 \quad \text{since} \quad w_C > w_S$$

The first result of decrease of subsistence sector goods supply might also bring a hike of its prices if its decrease of demand exceeds its decrease of supply.⁸ It's because the elasticity for subsistence sector goods against both income and prices is rather small. It, however, depends on the amount of labor shift since marginal labor productivity at the subsistence sector is assumed as nearly zero. Following figure reports an image of this effect of labor shift on its marginal productivity.

⁸ This implies the possibility that $\frac{\partial \pi}{\partial L_C} > 0$ could be satisfied.

Figure: Relationship between Labor and Marginal Productivity/Output at Subsistence Sector



Source: Author.

On the other hand, also suppose the capital stock is constant in a short run, price level of subsistence goods measured by capitalist goods (*i.e.*, p) and labor at capitalist sector (*i.e.*, L_C) are decided under the condition of the first equation of (EQ2) and that of (EQ7). At the above equations, other factors than these two variables such as p and L_C are not variables but intercept, which implies that they are decided according to functions of p , L_C and K . Differentiating the first equation of (EQ2) and that of (EQ7), we then obtain following equations:

(EQ9) results of differentiation

$$\frac{\partial^2 f}{\partial L_C \partial K} = \frac{\partial^2 f}{\partial L_C^2} \cdot \frac{dL}{dK} - \frac{\partial f}{\partial L_C} \cdot \frac{dp}{dK}$$

$$\frac{\partial \pi}{\partial L_C} \cdot \frac{dL_C}{dK} - \frac{dp}{dK} = 0$$

And solving above two equations of (EQ9) on $\frac{dp}{dK}$, we then obtain the results that capital accumulation has a positive differential coefficient on subsistence sector goods prices under the condition of sizable labor shift⁹ as follows:

(EQ10) effect of capital accumulation on subsistence goods price

$$\frac{dp}{dK} = \frac{-\frac{\partial \pi}{\partial L_C} \cdot \frac{\partial^2 f}{\partial L_C \partial K}}{\frac{\partial^2 f}{\partial L_C^2} - \frac{\partial f}{\partial L_C} \cdot \frac{\partial \pi}{\partial L_C}} > 0$$

since $\frac{\partial^2 f}{\partial L_C \partial K} > 0$, $\frac{\partial^2 f}{\partial L_C^2} < 0$ and $\frac{\partial f}{\partial L_C} > 0$

And also solving them on $\frac{dL_C}{dK}$, we then obtain the results that capital accumulation, of course, has a positive differential coefficient on labor at capitalist sector as follows:

(EQ11) effect of capital accumulation on labor at capitalist sector

$$\frac{dL_C}{dK} = \frac{-\frac{\partial^2 f}{\partial L_C \partial K}}{\frac{\partial^2 f}{\partial L_C^2} - \frac{\partial f}{\partial L_C} \cdot \frac{\partial \pi}{\partial L_C}} > 0$$

since $\frac{\partial^2 f}{\partial L_C \partial K} > 0$, $\frac{\partial^2 f}{\partial L_C^2} < 0$ and $\frac{\partial f}{\partial L_C} > 0$

⁹ Here, 'sizable labor shift' means that it satisfies $\frac{\partial \pi}{\partial L_C} > 0$.

These mathematical equations naturally represent that capital accumulation at capitalist sector leads to follows:

- (1) hike of subsistence sector goods prices.
- (2) increase of labor at capitalist sector.

Moreover, focusing on the first equation of (EQ3), we then find out follows:

(EQ12) investment or capital accumulation

$$\frac{dK}{dt} = rK - \mu K = \frac{\partial f}{\partial K} K - \mu K$$

or

$$\frac{\frac{dK}{dt}}{K} = \frac{\partial f}{\partial K} - \mu$$

The left hand of the second equation of (EQ12) reports growth rate of capital accumulation. When the benefit rate or the marginal productivity of capital exceeds the depreciation rate,¹⁰ the capitalist sector grows naturally. However, since it is also usually assumed that $\frac{\partial^2 f}{\partial K^2} < 0$, capital accumulation stops after marginal productivity of capital infers depreciation rate. The biggest problem is when it stops, after labor has shifted enough to pull up wage rate at subsistence sector to exceed the subsistence level or before?¹¹

Another important viewpoint is labor, or population that strongly correlates labor. The higher the growth rate of population is, the more labor shift from the subsistence sector to capitalist sector is required for pulling up marginal productivity of labor at the subsistence sector or ‘take-off.’¹²

5. Conclusion

This theoretical model for economic development is reported in a very preliminary manner. A further investigation would be required. One of the most required viewpoints for expansion of the model is to include a foreign sector that might provides net import (or aid) and foreign direct investment to a closed economy.

Apart from this, I have to point out that following policy implications appear important for economic development or ‘take-off.’

¹⁰ Of course, this implies that $\frac{\partial f}{\partial K} > \mu$.

¹¹ Of course, when labor shift pull up wage at subsistence sector up to its marginal productivity, this sector is no longer subsistent. Due to assumptions and definitions of Lewis model, this economy has already succeeded in ‘take-off.’ Moreover, from a historical viewpoint, no economy has so far reached at this stage that capital accumulation ceases.

¹² Yoshioka (2001) reports some simulation results of macroeconomic model that include insightful viewpoints of relationship between population and *per capita* GDP.

- (1) promotion of capital accumulation, in particular, that of high productivity.
- (2) encouragement of smooth labor shift from subsistence sector to capitalist sector under the modest population growth.

We can also derive some implication for Official Development Assistance (ODA) from developed countries to developing nations as follows:

- (1) ODA must help improvement of investment circumstances for promoting capital accumulation in developing countries.
- (2) ODA is also required to the subsistence sector to prevent decrease of its output when or after sizable labor shift¹³ begins. ODA for subsistence sector before it might discourage autonomous and smooth labor shift from the subsistence sector to the capitalist sector.

¹³ Here, I use 'sizable labor shift' such that brings decrease of subsistence sector goods output. Find out the difference with footnote 6.

(references)

- Banerjee, Abhijit V. and Andrew F. Newman (1996) "A Dual-Economy Model of Modernization and Development," *Development Discussion Paper No. 531*, Harvard Institute for International Development, Harvard University, 1996
- Emerson, Patrick M. (2002) "Corruption and Industrial Dualism in Less Developed Countries," *Journal of International Trade and Economic Development 11(1)*, 2002, pp.63-76
- Freeman, Richard B. and David L. Lindauer (1999) "Why Not Africa?" *NBER Working Paper No. 6942*, 1999
- Goodfriend, Marvin and John McDermott (1995) "Early Development," *American Economic Review 85(1)*, 1995, pp.116-33
- Jorgenson, Dale W. (1961) "The Development of a Dual Economy," *Economic Journal 71*, 1961, pp.309-334
- Leamer, Edward E. (1987) "Paths of Development in the Three-Factor, n-Good General Equilibrium Model," *Journal of Political Economy 95(5)*, 1987, pp.961-99
- Lewis, W. Arthur (1954) "Economic Development with Unlimited Supply of Labor," *The Manchester School of Economic and Social Studies 22*, 1954, pp.139-91; reprinted in A. N. Agarwala and S. P. Singh (eds.) *The Economics of Underdevelopment*, Oxford University Press, 1963
- Mankiw, N. Gregory, David Romer and David N. Weil (1992) "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics 107(2)*, 1992, pp.407-437
- Rappaport, Jordan (2002) "A Bottleneck Capital Model of Development," *RWP01-10*, Federal Reserve Bank of Kansas City, October 2002 (revised version)
- Sen, Amartya K. (1996) "Peasants and Dualism with or without Labor Surplus," *Journal of Political Economy 74*, 1966, pp.425-450
- Solow, Robert M. (1956) "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics 70*, 1956, pp.65-94
- Yoshioka, Shinji (2001) "Structure, Property, and Simulation Results of TSQ Model: New Version of TSQ Model QM0111," *TSQ Discussion Paper 2001/2002 No.6*, JICA-BAPPENAS-TSQ, 2001